GENERAL ARTICLE

Drawing in Mathematics

From Inverse Vision to the Liberation of Form

GEMMA ANDERSON, DOROTHY BUCK, TOM COATES AND ALESSIO CORTI

ABSTRACT

The literature on art and mathematics has focused largely on how geometric forms have influenced artists and on the use of computer visualization in mathematics. The authors consider a fundamental but undiscussed connection between mathematics and art: the role of drawing in mathematical research, both as a channel for creativity and intuition and as a language for communicating with other scientists. The authors argue that drawing, as a shared way of knowing, allows communication between mathematicians, artists and the wider public. They describe a collaboration based on drawing and "inverse vision" in which the differing logics of the artist and the mathematician are treated on equal terms.

Thinking is really the same as seeing.

—WILLIAM THURSTON, MATHEMATICIAN, 1946–2012

There has been much written, in the pages of *Leonardo* and elsewhere, on the connections between mathematics and art—on mathematical forms in works of art and as aesthetic objects, on the influence of higher-dimensional and non-Euclidean geometries on many artists, and on computer visualization in both mathematics and art [1]. We address here a vital gap in this discussion, describing the role of drawing in the *practice* of mathematical research and in the mathematical creative process. We argue that the shared roles of drawing in mathematics and the visual arts—drawing as a fundamental mode of understanding, and drawing as language—make possible a new mode of collaboration between artists and mathematicians, in which the different logics of the two disciplines coexist on equal terms. Here we describe our own collaboration, in which drawing-based dialogue and drawing as a way of knowing play essential roles. For the artist (Anderson), the collaboration provides access to beautiful and otherwise inaccessible geometries and the opportunity to experience and transform them, integrating them into her knowledge of form. For the mathematicians (Buck, Coates and Corti), the collaboration allows a new form of creativity, giving material form to purely conceptual objects, and brings their research to a wider audience in a nondidactic way. Together we develop a new visual vocabulary, drawn directly from contemporary mathematical research but stripped of all technical meaning. The mathematical forms and geometries are thereby liberated—freed from their original context and open to new understandings and interpretations.

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LINEAR LOGICAL THINKING AND MATHEMATICAL PROOF

We begin by introducing, by means of an example, a mode of thought that is fundamental to mathematical practice. This is what we call *linear logical thinking*. The paradigmatic example of linear logical thinking is the mathematical proof. This example's importance will be made clear below, as we describe mathematical creativity as a dialogue between two characters: the Thinker and the Drawer. The Thinker exists in the world of linear logical thinking. The Drawer operates in the world of the imagination and of inverse vision.

A proof in mathematics is a logical demonstration that some statement is true. Starting from something that is known to be true, we make a sequence of deductions, each following unassailably from the step before it, which leads to the desired statement. We illustrate this with the famous Theorem of Pythagoras:

> In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle [2].

In other words, if the sides of a right-angled triangle are of lengths a, b and c, as shown in Fig. 1(b), then $a^2 + b^2 = c^2$. Let us prove this [3].

Consider a square with side-length $a + b$, partitioned as

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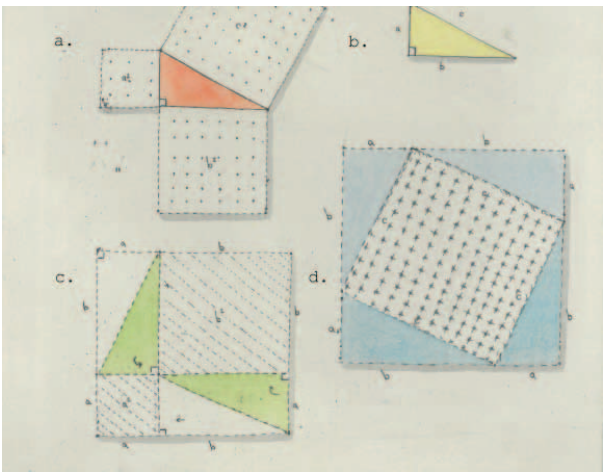

Fig. 1. Gemma Anderson, a drawing explaining the Theorem of Pythagoras. (© Gemma Anderson)

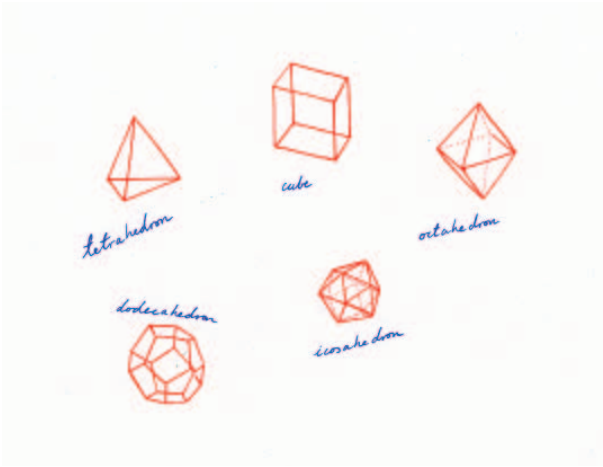

Fig. 2. Gemma Anderson, drawing of the five Platonic solids. (© Gemma Anderson)

shown in Fig. 1(c). One shaded square (the smaller one as shown) has side-length a, hence area a^2. The other shaded square has side-length b, hence area b^2. The total shaded area is therefore equal to $a^2 + b^2$. Each of the four triangles is right-angled, and the sides adjacent to the right angle have lengths a and b. It follows that in each case the hypotenuse (the side opposite to the right-angle) has length c. Thus, each of the four triangles is a copy of that shown in Fig. 1(b).

Now consider the same square partitioned in a different way as shown in Fig. 1(d). Once again, each of the four triangles is right-angled with sides a, b and c and thus is a duplication of the triangle in Fig. 1(b). Therefore, the length of each side of the shaded square is c, and so the area of the shaded square is c^2. Yet the total shaded area shown in Fig. 1(c) must be equal to the total shaded area shown in Fig. 1(d), for they are each equal to the area of the large square (a square of side-length $a + b$) minus the area of four copies of the triangle from Fig. 1(b). It follows that $a^2 + b^2 = c^2$.

DRAWING AS INVERSE VISION

In this paper, we focus on the drawing of imaginary objects—that is, objects that we see with our mind's eye. Whether the drawn object be physical or imaginary, all drawing is a sort of *inverse vision* [4]. By drawing with pencil on paper we give physical form to our mental images, and in the process we learn to see them better. Thus, in this context, drawing is a tool to train ourselves to see imaginary things better.

We illustrate this by showing some drawings made by Alessio Corti as a teenager. There are five regular (or Platonic) three-dimensional solids in Euclidean geometry (Fig. 2). Having read somewhere that in four dimensions [5] there are six regular solids and that one of them is made of 120 regular dodecahedra, Corti tried to prove this fact using the Euler formula in four dimensions, but he could not do it. Eventually he decided that the only way for him to prove the existence of this 120-cell was to draw it (Fig. 3).

The imaginary objects that are seen with inverse vision are visual objects: They may not have material form, but nevertheless they can be visualized. Thus, the drawing that we speak of here is the drawing of *visual objects*. There has been much written about the visual and spatial representation of *nonvisual* scientific objects, for example on the visualization of statistical data or on the visual representation of processes and of relationships between concepts [6]. These involve quite different forms of drawing, and we do not consider them here.

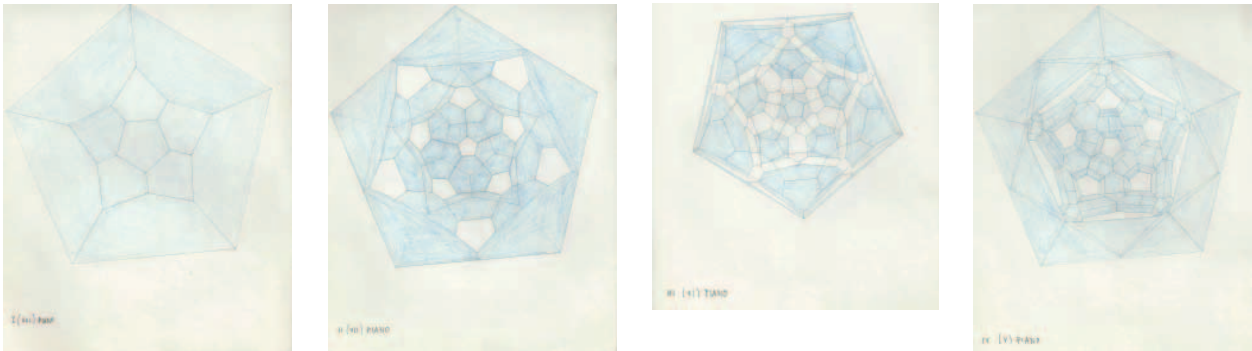

Fig. 3. Alessio Corti, sketches of a four-dimensional solid, the 120-cell, made of 120 dodecahedra. The 120-cell is constructed in layers. One begins with one dodecahedron (I) and then successively adds more dodecahedra by gluing along pentagonal faces (II, III). Image IV is halfway through the construction: 60 dodecahedra have been added. (© Alessio Corti)

DRAWING IN MATHEMATICAL CREATIVITY

Having said a little about what we mean by *thinking* and what we mean by *drawing*, we are ready to examine the use of drawing in mathematical research as a channel for intuition and creativity. This use of drawing is hidden—rarely spoken about among mathematicians and undiscussed in the literature on drawing. This absence of discussion is surprising, given how widespread the practice is in research mathematics, even in those sub-fields that frown on drawing-based proofs.

New mathematics does not start life as perfectly formed, rigorously proved theorems. A fundamental part of the creative process in mathematics is the passage from intuitive, imaginative understanding to rigorous, formal proof. This process is not a one-way transformation: We can think of it as a dialogue between two characters. Let us call them the Thinker and the Drawer. As we have noted, the Thinker operates in the world of linear logical thinking and of mathematical proof, while the Drawer operates in the world of the imagination and of inverse vision.

On the one hand, drawing gives the Thinker a way to organize thoughts. By listening to the Drawer, the Thinker is led to choose a sequence of logical steps that reflects the Drawer's inner vision. Thus the Drawer helps the Thinker to overcome otherwise unmanageable complexities. On the other hand, the Thinker's geometric rigor [7] helps the Drawer to focus and sharpen the inner vision, and the dialogue continues.

To make this concrete, consider the drawings and text in Fig. 4. Figure 4 depicts a calculation of a certain transformation called a "monodromy transformation," which measures the twisting of a shape as one moves around a loop in the parameter space for that shape. Here the monodromy transformation is given by the matrix (table of numbers) labeled T in Fig. 4 (bottom). The drawings and text are the last of a sequence of drawings, each recording a few exchanges in the dialogue described above. The process is incremental and iterative: One has a first go at it, makes a mistake and then has a second go, and so on. Each drawing (and calculation) in the sequence is better and truer to its object than the previous one. The final outcome, shown in the progression of drawings in Fig. 4, can be read in two different ways: as a visualization of the monodromy transformation and as a proof that the monodromy transformation is indeed given by the matrix T.

There is an interesting point of methodological similarity between Anderson's artistic process and this aspect of drawing in mathematics [8]. Even when drawing naturalistically, Anderson, like mathematicians, draws abstract objects—that is, objects that she sees with her mind's eye. For her, drawing often functions as a prelinguistic and premathematical form of intuition and abstraction, in which drawn objects, although often derived from the observable world, become abstract forms. This is in close parallel to the role of the Drawer in the mathematical creative process.

The use of drawing to channel mathematical intuition shares another, more essential aspect with Anderson's artistic process. In each case, the process of drawing and the reflection that accompanies it *transforms the drawer*,

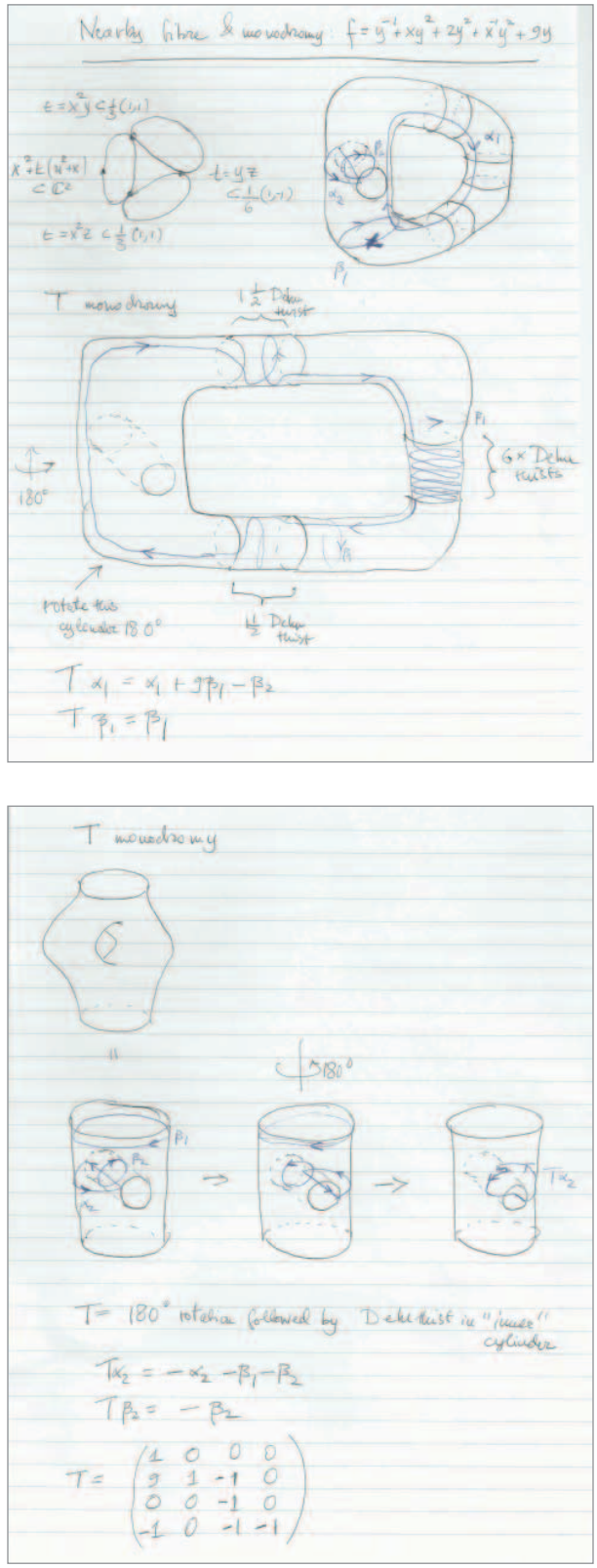

Fig. 4. Alessio Corti, drawings that convey mathematical intuition. (© Alessio Corti)

changing the way in which they know and understand the object that is being drawn. This explains why drawing is so much more effective in this context than the use of computer-based visualization software: It is the act and experience of drawing itself that creates intuitive understanding.

DRAWING AND COMMUNICATION

We have discussed the role of drawing in mathematical intuition and in the translation of that intuition into formal mathematical argument. Drawing also plays a key role in the communication of intuition between scientists, both between mathematicians and as part of interdisciplinary research.

Dorothy Buck is a topologist (a mathematician who studies shape and space) and also a mathematical biologist. Topologists sketch freely, both to develop their own intuition and to communicate that intuition to others. Figure 5, which depicts drawings produced by Buck and a colleague, illustrates an example of this. Buck and her colleague produced these drawings together, while conversing, each adding to or altering the drawing in turn, to try to understand how two complicated surfaces intersect.

These drawings are far less precise than the text and professional vocabulary in a typical mathematical research article, but they allowed Buck and her colleague to share and develop the essence of the ideas involved. In fact, the lack of precision here is an advantage: The ability to highlight mathematically interesting aspects while suppressing unimportant detail makes drawing a more useful tool than, for example, faithful computer-generated imagery.

Drawing is also essential in communication between topologists and molecular biologists. Because the professional vocabulary of both of these fields is highly technical, and because topologists and molecular biologists typically share no technical training, drawing serves as a vital bridge between the two disciplines. Drawing is the first language for developing questions and ideas; indeed, sharing intuition in mathematical-biological collaboration may not be possible in any other way.

For example, Buck and her molecular biologist collaborators often consider how DNA molecules become knotted and linked during cellular reactions such as replication and recombination (see Color Plate D and Fig. 6). Rather than introducing technical vocabulary and defining many terms, they draw how the central DNA axis twists and deforms during these reactions, thus immediately communicating the essential information.

THE LIBERATION OF FORM

We turn now to our ongoing collaboration and the artwork that we have made together. The collaboration began in February 2011, when Anderson saw the images in the Imperial College Newsletter article "A Periodic Table of Shapes" [9]. This article introduced Coates and Corti's research program, which concerns geometric forms called Fano Varieties. Fano Varieties are atomic pieces of mathematical shapes, and the goal of Coates and Corti's program is the classification of Fano Varieties in four dimensions [10]. Figure 7 consists of reproductions of some of the images originally contained in that article. The precise definition of a Fano Variety—provided in the Glossary—is a technical matter in mathematics

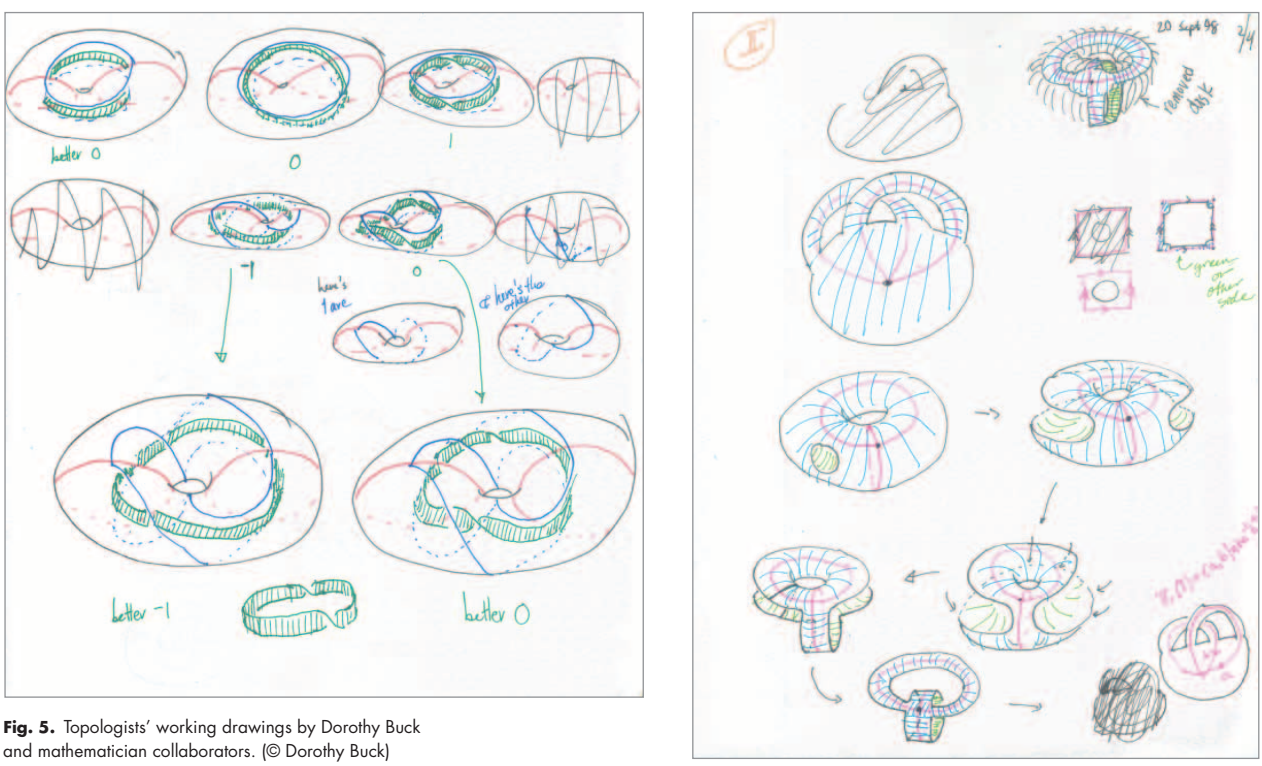

Fig. 5. Topologists' working drawings by Dorothy Buck and mathematician collaborators. (© Dorothy Buck)

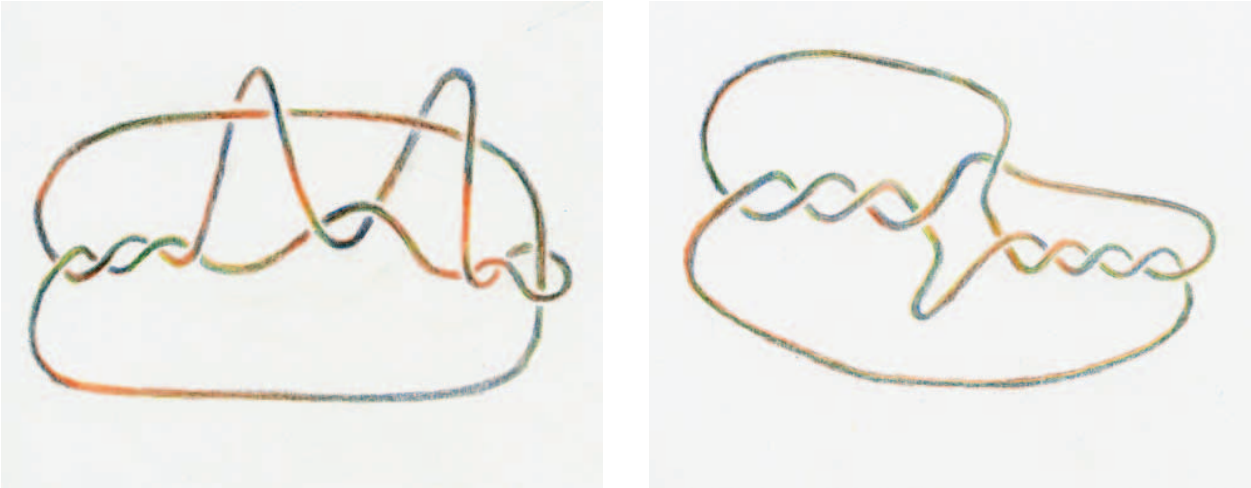

Fig. 6. Gemma Anderson, two drawings of the same mathematical knot. (© Gemma Anderson)

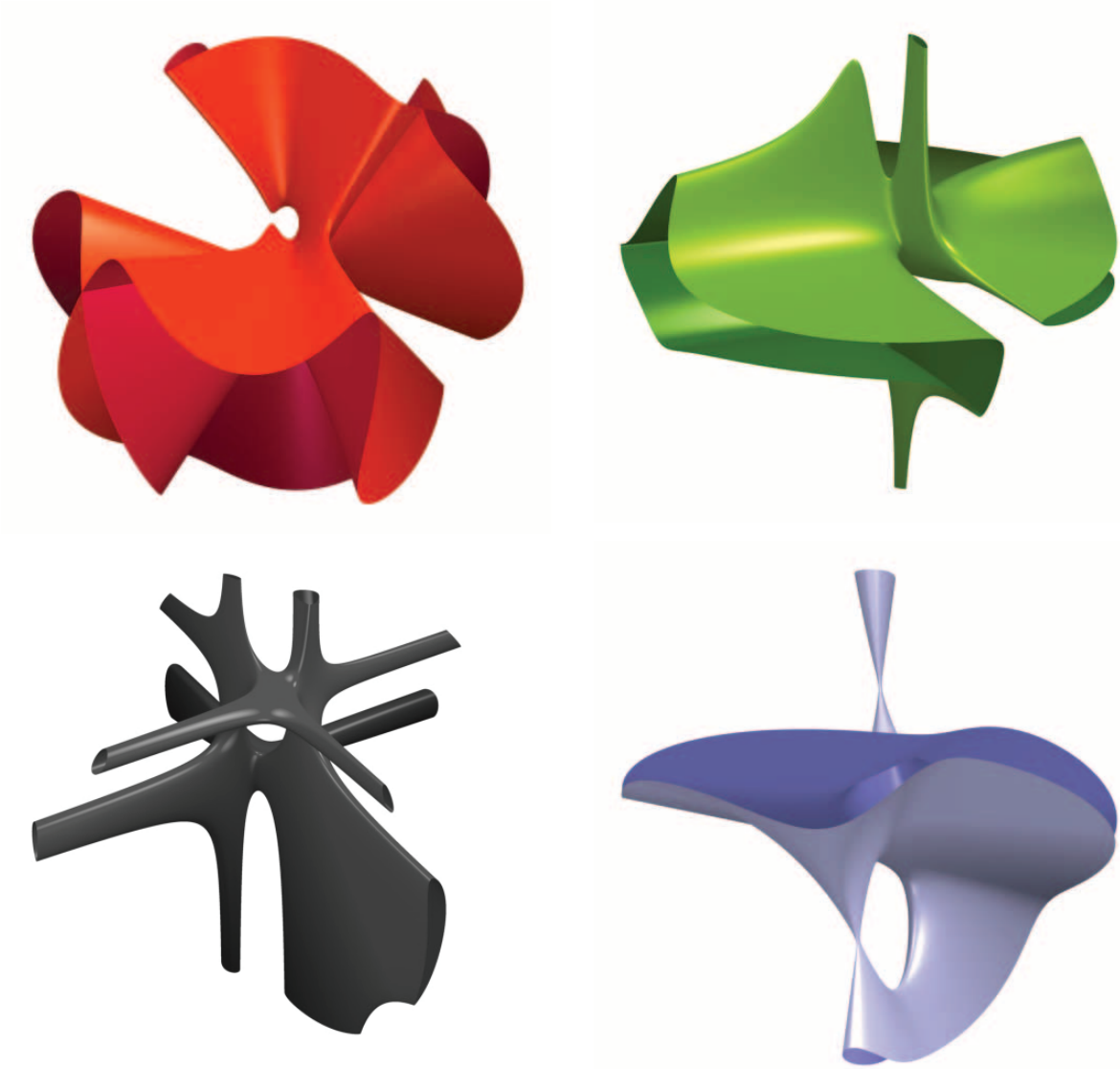

Fig. 7. Some three-dimensional Fano Varieties. (© Gemma Anderson)

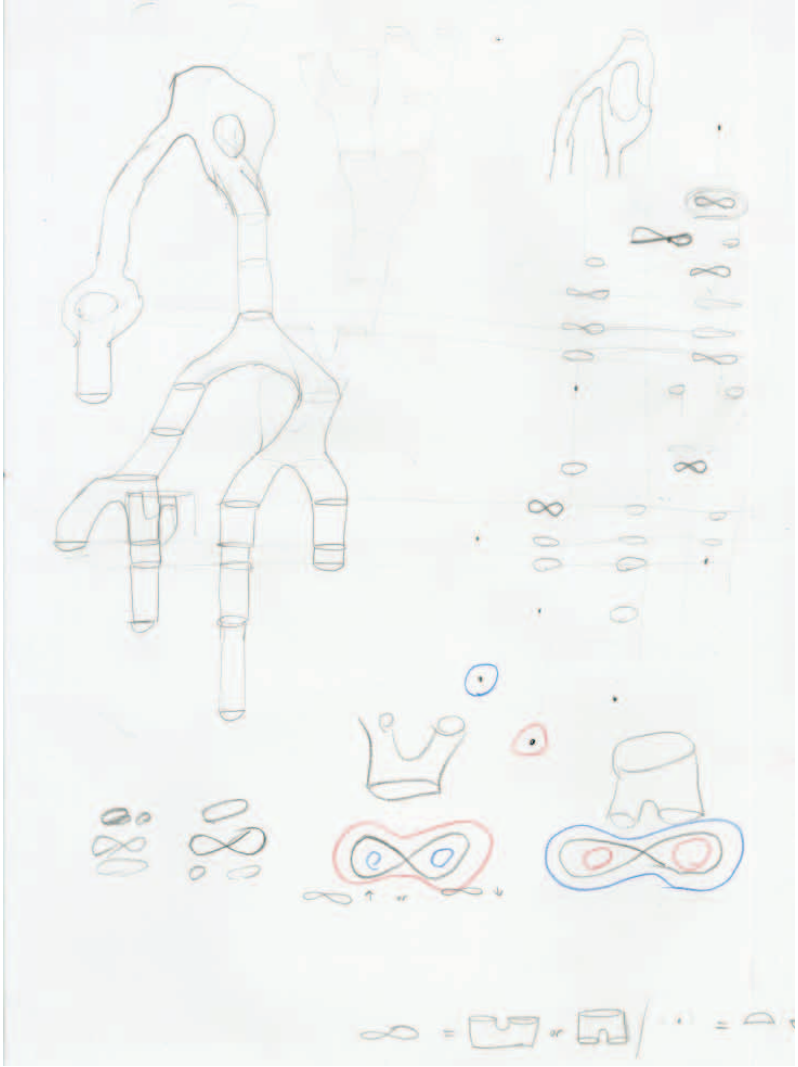

Fig. 8. A collaborative drawing made by Anderson and Corti when discussing Morse Theory, a part of topology. (© Gemma Anderson)

and is not so important here. We encourage the reader to look at the images in Fig. 8 purely as *images*, just as Anderson did, and not as mathematics.

Anderson is a visual artist whose practice is crucially informed by a long-standing interest in drawing and classification in the natural sciences. She was initially attracted to the unfamiliar, mysterious forms of the Fano Varieties purely as images. She was fascinated by the sense that these geometries exist in a world outside the physical world, and she was struck by the fact that mathematicians still lack (after almost a century of effort) a satisfactory system of classification for these forms. She took the article back to her studio and began to make drawings, exploring the forms, merging them and organizing them in different ways of her own.

Later this developed into a full-fledged collaboration, first between Anderson and Coates and then involving all of us. Our collaboration is founded on a shared commitment to drawing as a channel for creativity and imagination and on a shared faith in the intrinsic power of visual images to reach out to the viewer (artist, scientist or member of the public) in a direct and unmediated way. Drawing, too, serves as a privileged tool of communication between us (Fig. 8), allowing us to discuss the essential core of many mathematical and biological ideas despite the fact that we have quite different backgrounds; indeed Anderson has no formal scientific training at all. Our collaboration is still developing and is still very much experimental in character. In what follows, we briefly describe some of the artwork that we have made together and the techniques that we employed. There are two strands to the collaboration: One is centered around the Fano geometries and the other around knots and DNA.

Initially Anderson made an etching of all the rank-1 Fano Varieties, classified by shape and resemblance to one another (Fig. 9). We then made models of Fano Varieties as interlocking paper sculptures called sliceforms (Fig. 10) by using 3D printing and casting (Fig. 11). To build the models, we had to develop new software and algorithms for creating sliceforms from algebraic equations and also for turning these equations into the thickened polygonal meshes required for rapid prototyping. The algebraic equations on which the etchings and models are based were developed in Coates and Corti's research program. We used the open source program surfex to visualize and adjust certain parameters. We used new code written in Mathematica to generate the sliceforms; we also used Mathematica, as well as the open source program MeshLab and new code written in the new open source mathematical software Sage, to turn the equations into .stl and .obj files suitable for 3D printing [11]. In building the Fano models we make contact with a long tradition of mathematical model building in the 19th century [12], now largely lost, but we revisit this with the full power of 21st-century mathematical science [13] and with a blend of traditional and modern techniques: etching, hand-tinting, casting, rapid prototyping, laser cutting, computer algebra and computer-aided manufacture.

In our work on knots, Anderson again began by experimenting, drawing different diagrams of the same knot (see Fig. 8) and different knots. During this process, Buck and Anderson discussed how DNA might be represented and how selectively highlighting features may aid our understanding of these knotted molecules. Anderson then created 3D DNA knots in different media. For example, she made ceramic knots and links, some with helices as part of their structure, and used a variety of glazing and sketching techniques to highlight the molecular structure. Anderson and Buck together created a detailed drawing of a section of DNA, which Anderson then wove as a jacquard textile

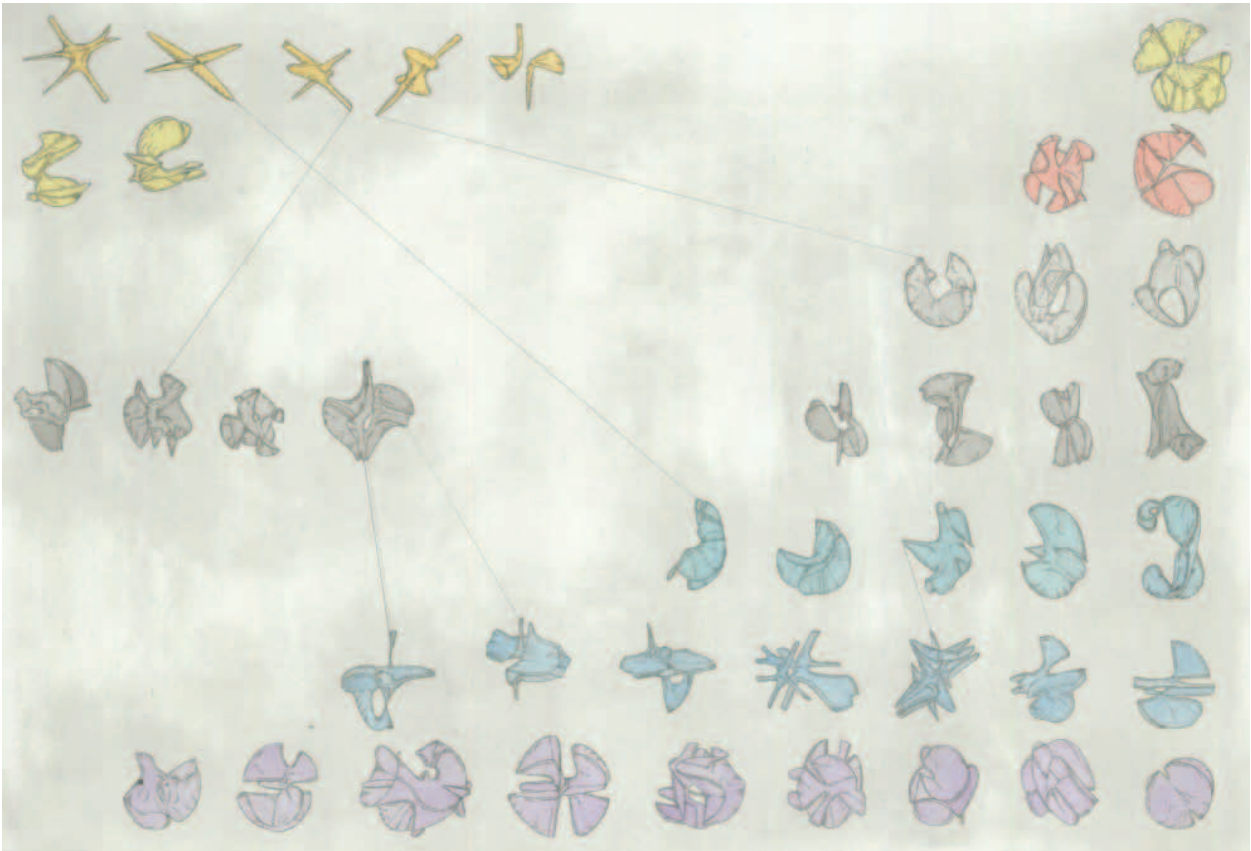

Fig. 9. Gemma Anderson, *Periodic Table of Fano Varieties*, copper etching and watercolor, 2012. (© Gemma Anderson)

Fig. 10. Gemma Anderson and Tom Coates, *Sliceform*, three-dimensional Fano Variety model consisting of interlocking two-dimensional, laser-cut slices, 2012. Each paper slice that makes up the sliceform is etched with images, originally created as copper etchings made by Anderson, related to that Fano Variety. A laser was used to cut these images directly onto the paper. (© Gemma Anderson. Photo: Nick White.)

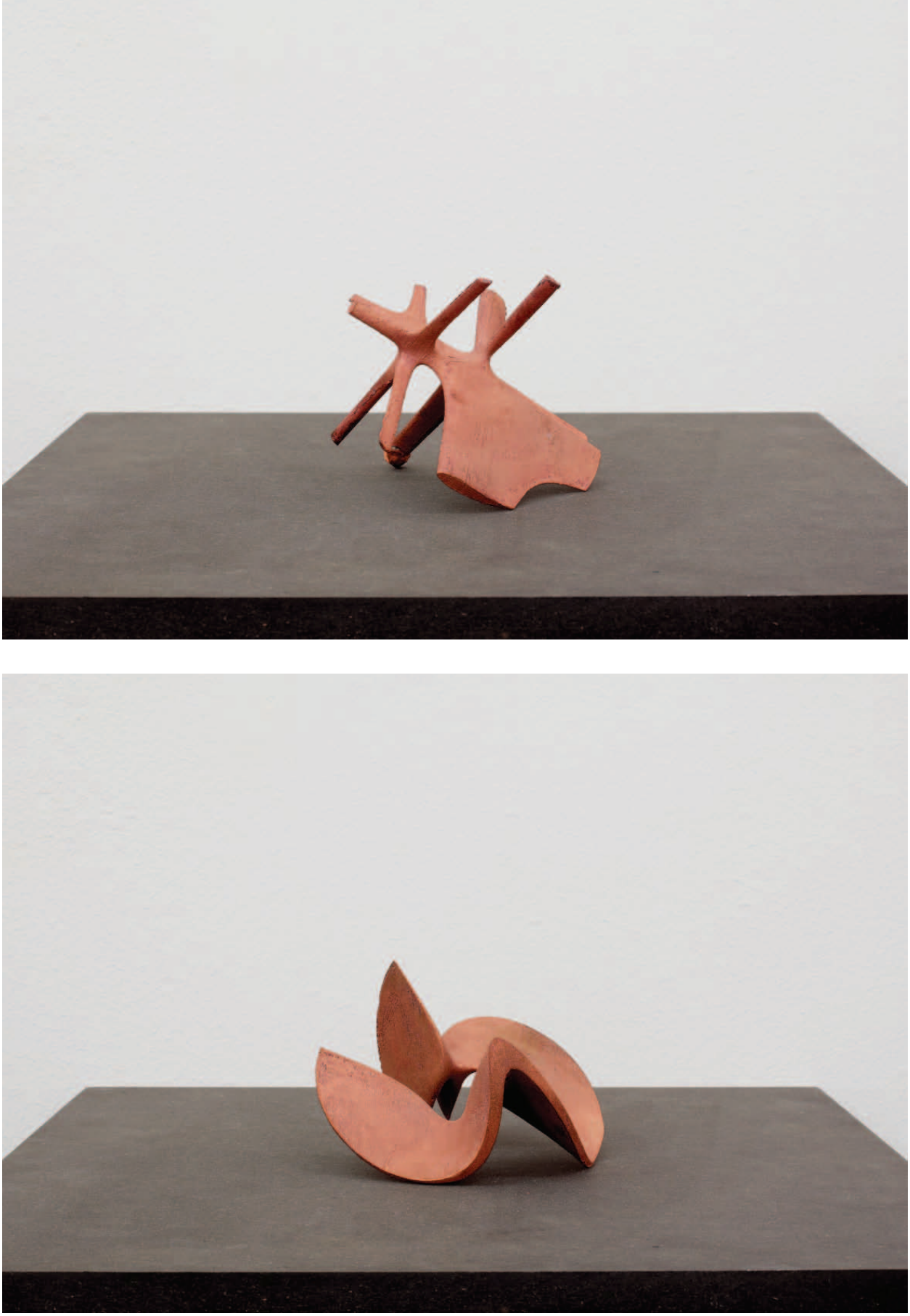

Fig. 11. Gemma Anderson and Tom Coates, 3_{27} and B_4, copper investment casts of RP forms, 2012. (© Gemma Anderson. Photos: Nick White.)

that can be knotted or linked. Scientists routinely visualize knotted and linked DNA molecules through computational simulations (using Metropolis Monte Carlo or Molecular Dynamics methods). This collaboration has produced some of the first artistic representations of these forms.

OUR WORK IN CONTEXT

The advances in computer technology that made possible the Visual Mathematics movement at the end of the 20th century also make possible the first step in our collaborative process: shared visualization. Yet the work we produce is neither Visual Mathematics nor, as Max Bill has called for, "art based on the principles of mathematics" [14,15]. On the other hand, there is a long history of artists incorporating mathematical forms into their artistic vocabulary and transforming them through their practice—for example, the influence of non-Euclidean geometries on the Russian avant-garde [16]. Our work continues this tradition, using contemporary computer visualization technology only as an intermediate, if essential, part of our process.

The drawings and models that we have made as artworks are quite different from the drawings that we discussed above. In mathematics, drawing is typically an informal process, which is later translated into algebra or text. In Anderson's process there need be no further translation: The drawing is the work.

Throughout the collaboration, our creative process has been almost entirely guided by Anderson. Allowing themselves to be guided by Anderson's artistic vision, the mathematicians in the collaboration gained some unexpected benefits. By giving up the traditional, didactic approach to scientific popularization, they did not have to infantilize, compromise or falsify their ideas. The artworks that we produce are true to the mathematical objects they represent, even as they carry none of the technical context where those objects originate. The Fano models and knot sculptures give body and weight to forms that had no previous physical existence, and it is precisely by giving these forms a body that is stripped of the original scientific meaning that we can bring contemporary mathematical research to a wider audience in a direct and unmediated way. Through drawing and modeling, the forms are liberated and can exist and function on different levels [17]. No longer constrained by their mathematical meaning, they become accessible to different forms of understanding and appreciation by artists, scientists and the wider public.

Acknowledgments

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References and Notes

1 See for example: C. Johnson, "On the Mathematics of Geometry in My Abstract Paintings," *Leonardo*, Vol. 5, No. 2 (1972) pp. 97–101; C.A. Pickover, "Mathematics and Beauty: A Sampling of Spirals and 'Strange' Spirals in Science, Nature and Art," *Leonardo*, Vol. 21, No. 2 (1988) pp. 173–181; *Leonardo*, Vol. 25, No. 3/4, Visual Mathematics: Special Double Issue (1992); *The Visual Mind: Art and Mathematics*, edited by M. Emmer, Leonardo Book Series (Cambridge, MA: MIT Press, 1993); J. Melvin and R. Hewison, "From Ruskinian Drawing Exercises to Advanced Mathematics—With Architecture, Painting and Sculpture in Between—Representation of Ideas and Objects Lies at the Heart of Intellectual Endeavour," *The Architectural Review*, Vol. 217, No. 1296, February 2005; B. Kalantari, "Polynomiography: From the Fundamental Theorem of Algebra to Art," *Leonardo*, Vol. 38, No. 3 (2005) pp. 233–238; *The Visual Mind II,* edited by M. Emmer, Leonardo Book Series (Cambridge, MA: MIT Press, 2005); J. Mandelbrojt, "Similarities and Contrasts in Artistic and Scientific Creation-Discovery," *Leonardo*, Vol. 39, No. 5 (2006) pp. 420–425, 435; *Mathematics and the Aesthetic: New approaches to an ancient affinity*, edited by N. Sinclair, D. Pimm and W. Higginson (New York, NY: Springer, 2006).

2 T.L. Heath, *Euclid: The Thirteen Books of the Elements,* Dover, 1956.

3 This proof, which we gave as an example of linear logical thinking, is drawing based. Drawing-based mathematical proofs are rare, and indeed only occur in certain sub-fields of mathematics. The types of argument that are considered acceptable in a mathematical proof have varied throughout history and vary across the different parts of mathematics; drawing-based proofs are (and were) accepted only at some times and only in some contexts. There are delicate questions of mathematical philosophy lurking here—what *precisely* do we mean by *proof*? A lucid account of these issues, and a spirited defense of visual reasoning in mathematics, can be found in J.R. Brown, *Philosophy of mathematics: A contemporary introduction to the world of proofs and pictures,* Second edition (New York, NY: Routledge/Taylor & Francis Group, 2008). For a discussion of diagrams and visualizations as explanations, see also D.W. Henderson and D. Taimina, "Experiencing Meanings in Geometry," in *Mathematics and the aesthetic: New approaches to an ancient affinity*, edited by N. Sinclair, D. Pimm and W. Higginson (New York, NY: Springer, 2006).

4 See W.P. Thurston, "On Proof and Progress in Mathematics," *Bulletin of the American Mathematical Society*, Vol. 30, No. 2 (1994) pp. 161–177. Thurston writes: "People have very powerful facilities for taking in information visually or kinesthetically, and thinking with their spatial sense. On the other hand, they do not have a very good built-in facility for inverse vision, that is, turning an internal spatial understanding back into a two-dimensional image. Consequently, mathematicians usually have fewer and poorer figures in their papers and books than in their heads" (Ibid., p. 164).

5 For a mathematician, to say that a shape is two-dimensional means that each point of the shape can be described using two coordinates (x,y). For example, the surface of the Earth is two-dimensional because any point on the surface of the Earth can be represented by two numbers: latitude and longitude. The space in which we live is three-dimensional, because a point in that space can be represented by three coordinates (x,y,z). Similarly, points in four-dimensional space are described by four numbers (x,y,z,t). In Einstein's Theory of Relativity, the fourth coordinate "t" is taken to represent time, but in our context there is no need to insist on this. We cannot experience or perceive four-dimensional space in the same way that we do three-dimensional space, but nonetheless we can conceive of and reason about it.

6 See, for example, E. Tufte's magisterial work on statistical visualization (*The Visual Display of Quantitative Information*, Graphics Press, 1983; *Envisioning Information*, Graphics Press, 1990; *Visual Explanations: Images and Quantities, Evidence and Narrative*, Graphics Press, 1997); M. Lynch, "Discipline and the Material Form of Images: An

Analysis of Scientific Visibility," *Social Studies of Science, SAGE, London, Beverly Hills and New Delhi*, Vol. 15 (1985) pp. 37–66; T. Hankins, "Blood, Dirt, and Nomograms: A Particular History of Graphs," *Isis*, Vol. 90, No. 1 (1999) pp. 50–80.

7 See M. Goresky and R. MacPherson, *Stratified Morse Theory* (Berlin, Germany: Springer-Verlag, 1988) p. 22. Historically, in geometry there has always been a tension between imagination and rigor. *Stratified Morse Theory* is a major contribution to the notoriously ill-founded field of differential topology. Goresky and MacPherson develop drawing-based methods of mathematical proof and, in the Introduction, stress the importance of rigorization that is geometrically apt. Our own interpretation of this process is the dialogue between the Thinker and the Drawer.

8 This similarity undoubtedly holds for many other artists as well. But it certainly does not hold for all artists, as many contemporary visual artists simply do not draw.

9 "Periodic Table of Shapes," *Imperial College Reporter*, Vol. 230, 4 March 2011. "Periodic table of shapes to give a new dimension to maths," Imperial College London News and Events, 16 February 2011. <www3.imperial.ac.uk/newsandeventspggrp/imperialcollege/newssummary/news_16-2-2011-8-32-29>.

10 T. Coates, A. Corti, S. Galkin, V. Golyshev and A. Kasprzyk, *Mirror Symmetry and Fano Manifolds*, 2012, <www.arxiv.org/abs/1212.1722>.

11 S. Holzer, O. Labs, *surfex 0.90*. University of Mainz and University of Saarbrücken, 2006: <www.surfex.AlgebraicSurface.net>. Wolfram Research, Inc., *Mathematica*, Version 8.0, Champaign, IL, 2010. P. Cignoni, M. Corsini, G. Ranzuglia and *MeshLab: An open-source 3d mesh processing system*, ERCIM News (73), 2008, <http://meshlab.sourceforge.net>. W.A. Stein et al., *Sage Mathematics Software* (Version 5.1), The Sage Development Team, 2012, <www.sagemath.org>.

12 Anderson was inspired by mathematical models in the Science Museum, especially the cardboard sliceform models of ellipsoids made at the Munich Workshops taught by Felix Klein and Alexander Von Brill in the 1870s. Also of inspiration: a model of the cubic surface made by Olaus Henrici in 1875 and paper models from Joseph Albers' Bauhaus preliminary courses (1925–1928).

13 The classification questions that Coates and Corti study have been open since the work of Gino Fano in the 1930s, yet the techniques that they apply—a blend of ideas from geometry, string theory and high-performance computing—would have been unthinkable even a decade ago.

14 The mathematics that we describe here is not Visual Mathematics in the sense of Emmer [1]. Some of the authors' research makes heavy use of computers, to generate data and test conjectures, but we seldom use computer-based *visualizations* in our work: Computers are used to perform many thousands of algebraic manipulations and to solve equations, not to visualize the complex geometries that we study.

15 M. Bill, *The Mathematical Way of Thinking in the Visual Art of our Time*, in Emmer [1].

16 See M. Corrada, "On Some Vistas Disclosed by Mathematics to the Russian Avant-Garde: Geometry, El Lissitzky and Gabo," *Leonardo*, Vol. 25, No. 3/4, Visual Mathematics: Special Double Issue (1992), pp. 377–384.

17 "A work of art, therefore, is a complete and *closed* form in its uniqueness as a balanced organic whole, while at the same time constituting an *open* product on account of its susceptibility to countless different interpretations which do not impinge on its unadulterable specificity." U. Eco, *Opera Aperta,* Bompiano, Milan, 1962.

Glossary

Euler formula—in three dimensions: the equality V – E + F = 2, where V is the number of vertices of a three-dimensional solid, E is the number of edges, and F is the number of faces. In four dimensions: the equality V – E + F – T = 0, where V is the number of vertices of a four-dimensional solid, E is the number of edges, F is the number of faces, and T is the number of three-dimensional facets.

Fano Variety—Fano Varieties, roughly speaking, are "atomic pieces" of higher-dimensional geometrical shapes. In precise mathematical terms, they are smooth projective complex manifolds with ample anticanonical line bundle.

inverse vision—turning an internal spatial understanding back into a two-dimensional image: see [4].

linear logical thinking—the sort of reasoning that constitutes a mathematical proof.

monodromy—a measure of the twisting of a shape as you move around a loop in the parameter space for that shape.

parameter space—mathematical shapes can depend on "deformation parameters." Changing these parameters changes the sizes and relative positions of different parts of the shape but leaves the qualitative form of the shape unchanged. We can think of a single shape (with fixed deformation parameters) as giving a point in the space of all possible parameters, which is called parameter space.

topology—the mathematical study of shape.

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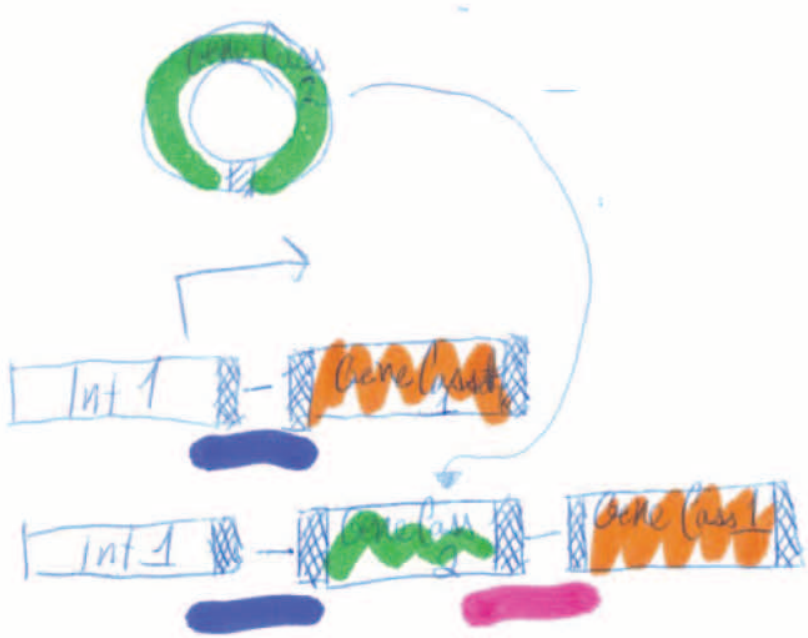
COLOR PLATE D: **DRAWING IN MATHEMATICS**



Working sketch by Dorothy Buck and biologist collaborators created while planning a series of experiments. This shows particular sections of DNA molecules that will later interact to form knots. Colors suggest the biological interaction: Matching colors denote that subsections will be joined. The rest of the DNA structure (other sections and more detailed molecular representations) is suppressed so that the knotting reaction is highlighted. (© Dorothy Buck) See article by Gemma Anderson et al.

Thanks to Our Reviewers

The Leonardo publications are peer reviewed: before acceptance, every article is reviewed and commented on, usually by three anonymous reviewers. Based on these reviews we make a decision whether to publish the text.

Leonardo covers a wide variety of fields, many of them interdisciplinary and often in emerging areas of experimentation, research and scholarship. Our peer reviewers help us focus on new topics of interest, as other areas may move out of the center of experimentation and become part of established disciplines. In very new areas of inquiry it is often difficult for us to identify three reviewers competent to review the work; in these cases we may ask the authors themselves to suggest new reviewers for our peer review panel. The Leonardo Network in a very real sense grows and evolves through this process of collaborative filtering.

We wish to take this opportunity to thank some of the reviewers who in the past year have on a volunteer basis spent time writing very thoughtful reviews that both help us make a publication decision and help the authors to improve their texts:

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